Electromagnetism Part 9: Magnetic Dipole Moment and Magnetisation

Mike Hughes

Contents

1 Introduction

In this section we cover the idea of the magnetic dipole moment and explore magnetisation. Again, we stick with magnetostatics, we only have constant currents.

2 Magnetic Dipole Moment

Consider a square loop of wire with a current flowing through it. Place it in a uniform magnetic field. As drawn, the top and the bottom of the loop experience no force, as the current is parallel with the magnetic field. The left and right sides experience forces in opposite directions.

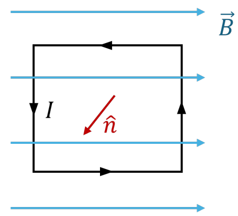

Figure 1: Torque on a square loop of wire in a magnetic field

There is therefore a torque on the loop. By analogy with the electric dipole moment ($\vec{p} = q\vec{d}$), we describe this as a magnetic dipole moment,

$$\vec{m} = I\vec{A}$$

where the vector A has magnitude equal to the loop area and points perpendicular to the loop (in the direction given by the right hand rule). Point your fingers on your right hand in the direction of the flowing current, and your thumb points in the direction of $\vec{A}$ and $\vec{m}$. $\vec{m}$ has units of Am^2.

The torque is then,

$$\tau = \vec{m} \times \vec{B}$$

and the dipole has potential energy,

$$U = -\vec{m} \cdot \vec{B}$$

These current loops occur naturally in all kind of places. As well as physical current loops, electrons have an intrinsic spin that acts like a current loop, as do the atomic nuclei. Electrons and nuclei therefore have magnetic dipole moments, which is very important in atomic and nuclear physics.

3 Types of Magnets

All magnetic phenomena are a result of charges in motion. In most cases the current loops are very small, such as electrons orbiting a nucleus. Within a material these effects usually cancel out, so there is no net dipole moment. But when we put materials in a magnetic field, there is a net alignment of the dipoles and the medium becomes magnetically polarised or **magnetised** (compare to *polarisation* by electric fields).

There are three distinct cases:

- **Paramagnets**: The magnetisation is parallel to $\vec{B}$.
- **Diamagnets**: The magnetisation is opposite to $\vec{B}$.
- **Ferromagnets**: The magnetisation is maintained even after $\vec{B}$ is removed.

Paramagnetism and diamagnetism are extremely weak, and so when we refer to magnets in everyday experience we usually just mean ferromagnetism.

4 Magnetisation

If matter is put in a magnetic field, it becomes **magnetised**. We define the magnetisation, $\vec{M}$ as the **magnetic dipole moment per unit volume**. Like polarisation, this a macroscopic property of the material, and we can often work with it without worrying about the microscopic details of what is actually happening.

Similar to how polarisation creates bound surface charge (and potentially bound volume charge if $\nabla \cdot \vec{P} \neq= 0$, the effect of magnetisation on matter is to create bound currents within the material,

$$\vec{J}_b = \nabla \times \vec{M}$$

and on the surface,

$$\vec{K}_b = \vec{M} \times \hat{n}.$$

The magnetisation therefore creates another magnetic field which is determined by these currents.

We might also have some free current flowing anyway, $\vec{J}_f$, and so the total current,

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

With polarisation, when we considered free and bound charge, we could then re-write Gauss's Law to reach the idea of displacement, $\vec{D}$. Let's do something similar for magnetisation, but starting with Ampere's Law,

$$\frac{1}{\mu_0}(\nabla \times \vec{B}) = \vec{J} = \vec{J}_f + \vec{J}_b = \vec{J}_f + (\nabla \times \vec{M})$$

which gives,

$$\nabla \times \left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \vec{J}_f$$

Now, we define,

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M},$$

and so we have,

$$\boxed{\nabla \times \vec{H} = \vec{J}_f}$$

or, in integral form,

$$\boxed{\oint \vec{H} \cdot d\vec{l} = I_{f,enc}}$$

So, like how $\vec{D}$ allows us to express Gauss's Law in terms of only the free charge, and not worry about the bound charges created due to polarisation, now we can express Ampere's Law in terms of only the free current, without worrying about the bound currents created by magnetisation.

$\vec{H}$ is in some ways more useful than $\vec{D}$. If we put an electric current through a wire then we know $\vec{J}_f$ but don't necessarily know anything about the magnetisation. So it's helpful to talk about $\vec{H}$, which we can calculate, rather than $\vec{B}$. On the other hand if you connect a piece of metal to a power supply with a fixed voltage, you end up fixing V and hence E regardless of the polarisation, so it is $\vec{D}$ that you don't know. So, sometimes you will see Maxwell's equations written using $\vec{E}$ and $\vec{H}$. And often $\vec{H}$ is referred to as the **magnetic field**, and $\vec{B}$ as the **magnetic flux density**, while at other times we often just call $\vec{B}$ the magnetic field.

Note that while $\nabla \cdot \vec{B} = 0$ (i.e, there are no magnetic monopoles), we have $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$.

5 Magnetic Susceptibility and Permeability

Just like with polarisation, if $\vec{M}$ is proportional to $\vec{H}$ (or in practice $\vec{B}$ since $B \gg H$) then we have a simplification. Note this is true (usually) for para and diamagnetism, but not ferromagnetism (which persists even when the field drops to zero!). We define the **magnetic susceptibility**, χ_M, by,

$$\vec{M} = \chi_M \vec{H}$$

Typical values of χ_M are 10^{-5}, and it is positive for paramagnets and negative for diamagnets. Then we have,

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_M)\vec{H},$$

and so,

$$\boxed{\vec{B} = \mu\vec{H},}$$

where $\mu = \mu_0(1 + \chi_M)$ is the permeability of the material. ($\mu = \mu_0$ in free space). Remember this is only true where the magnetisation is linear with $\vec{B}$, and definitely **not** for ferromagnets like iron.

6 Ferromagnetism

In ferromagnets, the magnetic dipoles tend to align naturally even when there is no $\vec{B}$ field applied. However, they only align over small regions, called domains, and so on a macroscopic scale there is no net magnetisation. When we put the ferromagnetic in a strong magnetic field, this tends to force the domains to merge, so we end up with some net magnetisation which persists even after the magnetic field is turned off, creating a permanent magnet. Heating a magnet to high temperatures (e.g. 770°C for iron, known as the Curie point) destroys the alignment and it returns to being simply a paramagnet.

Some ferromagnetic materials, such as soft iron, lose their magnetisation very quickly when the external magnetic field is turned off. This is why iron is used in the core of an electromagnet (with a solenoid wrapped around it). The iron becomes magnetised when the current is on, greatly increasing the magnetic field, by a factor of a few thousand times, i.e.

$$\vec{B} = \mu\vec{H} = \mu n I,$$

where I is the current in the solenoid and n is the number of terms, and μ ranges from 1000 to 60,000 depending on the field strength.