Electromagnetism Reference: Curvilinear Co-ordinates

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These are some useful definitions for curvilinear co-ordinate systems.

Coordinate Systems Elements

Cylindrical Coordinates (r, θ, z)

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad z = z$$

Volume Element

$$dV = r\,dr\,d\theta\,dz$$

Surface Element

$$dS = r\,d\theta\,dz\,\hat{r}$$

Spherical Coordinates (r, θ, ϕ)

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$

Volume Element

$$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

Surface Element

$$dS = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$

Differential Operators

Gradient

Cylindrical Coordinates

$$\nabla f = \frac{\partial f}{\partial r} \, \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \, \hat{\theta} + \frac{\partial f}{\partial z} \, \hat{z}$$

Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r} \, \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \, \hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi} \, \hat{\phi}$$

Divergence

Cylindrical Coordinates

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r}(r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

Spherical Coordinates

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(\sin\theta \, A_\theta) + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

Curl

Cylindrical Coordinates

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & rA_\theta & A_z \end{vmatrix}$$

Spherical Coordinates

$$\nabla \times \vec{A} = \frac{1}{r \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$